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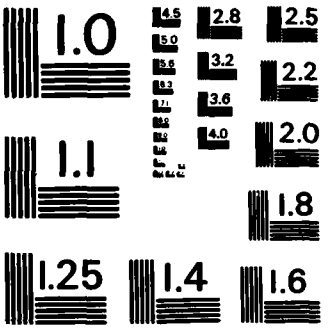

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PROGRESS REPORT

on

AFOSR GRANT NO. 82-0189

MARKOV PROCESSES

APPLIED TO CONTROL, REPLACEMENT, AND SIGNAL ANALYSIS

for the period

1 June 1983 - 31 May 1984

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Principle Investigator

E. CINLAR

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This is an account of the work accomplished during 1 June 1983 and 31 May 1984 under the grant AFOSR 82-0189.

A. DEFORMATION OF SOLIDS

The aim of this work is to build stochastic models for the nucleation and growth of microcracks in solids. This is intimately related to reliability of engineering materials, since crack nucleation and growth is the major mechanism leading to failure of machine parts.

Our work is concentrated on the early stages, where the cracks are still of atomic dimensions. In the paper "A model for the initiation of microcracks" we introduce a model for the phenomena under consideration. The model is a spatial pure birth process. Mathematically, it is a Markov process evolving over time; every state of the Markov process is a purely atomic measure; the atoms of the measures represent the cracks in the body. We were able to construct such a Markov process by using our other work on self-exciting point process. Then, we computed the infinitesimal generator of the process, wrote Kolmogorov differential equations for it, and computed the means etc. for various descriptive quantities associated with the configuration of cracks as it evolves over time. Two copies of this paper are attached.

B. RELIABILITY

We have had some exploratory work on the reliability of complex devices with many components. We concentrated on the problem of characterizing the dependence of the components' lifetimes on each other. This is generally caused by the fact that all the components are subjected to the same environmental conditions.

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On the same topic, principal investigator was one of the editors of a volume "Reliability Theory and Models" published by Academic Press. A copy of this volume is attached.

C. SEMINAR ON STOCHASTIC PROCESSES

The third seminar on stochastic processes was held in 1983 and brought together a number of distinguished researchers on stochastic processes. The volume "Seminar on Stochastic Processes 1983" was edited and is now published by Birkhäuser. A copy of this volume is attached.

D. TEXT ON STOCHASTIC PROCESSES

This is meant to be a text on stochastic processes assuming a one-year graduate course in probability as a background. It is to cover Poisson random measures, Brownian motion, Levy processes, martingales, stochastic integrals, and Markov processes. A rough draft of everything (except Markov process part, which was written and sent to AFOSR earlier) is attached. We hope to do a second version in the near future.

E. STABILITY OF DEPENDENT RANDOM VARIABLES

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This work was done by N. Bouzar who was supported by the grant as a post-doctoral fellow. It concerns the stability of sums $S_n = X_1 + \ldots + X_n$ of dependent random variables in comparison with the sums

$$S_n' = \sum_{i=1}^{n} E(X_i | \mathcal{H}_{i-1}) ,$$

where $\mathcal{H}_{i-1}$ is the information available at time i-1. In other words, S_n' is the estimate of S_n just prior to time n, and the question is about the difference $S_n - S_n'$: does the difference stabilize as $n \to \infty$. A paper

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written on this by Bouzar, "A note on a form of Borel-Cantelli Lemma" is to appear in the journal "Advances in Mathematics."

A second problem, investigated jointly by Bouzar, Austin, and Dvoretzky, concerns the finitely additive measures associated with an adapted sequence of integrable random variables. We shall report on it when this work is completed.

F. MEAN EXIT TIME OF BROWNIAN MOTION WITH DRIFT

This work was carried out by M. Pinsky. In previous work on this grant we have studied the mean exit time of Brownian motion from a small geodesic ball in a Riemannian manifold. In this work (M,g) is an n-dimensional Riemannian manifold and Δ is the Laplacian which is canonically associated with the metric g. The Brownian motion is a strong Markov process with infinitesimal generator Δ. The exit time from a ball is $T_\varepsilon^g = \inf\{t > 0 : d(X_t,m) = \varepsilon\}$ where d is the Riemannian distance and $m \in M$. It was found that if $n < 6$ the knowledge of the mean exit time $E_m(T_\varepsilon)$ for all $\varepsilon > 0$ determines the Riemannian metric g. In order to establish this we found an asymptotic expansion of the mean exit time for small $\varepsilon > 0$.

In our current work we study Brownian motion with drift, the diffusion process generated by $\frac{1}{2}\Delta + b \cdot \nabla$ where b is a smooth vector field on M. It is conjectured that the mean exit time $E_m(T_\varepsilon^b)$ for all b determines the metric g without regard to dimension. In order to prove this we develop the following expansion in a system of normal coordinates: $(\frac{1}{2}\Delta + b \cdot \nabla) = \frac{1}{2}\Delta_{-2} + (\frac{1}{2}\Delta_{-1} + B_{-1}) + \ldots$ where the correction terms to the Euclidian laplacian Δ_{-2} are second order differential operators with polynomial coefficients. A first consequence of this is the asymptotic

expansion $E_m(T_\varepsilon^b) = \varepsilon^2/n + \text{const.}\ \varepsilon^4(\tau_m - 6(\text{div } b)_m - 6|b|_m^2) + O(\varepsilon^6)$, which displays the interaction of the "force field" b and the Riemannian metric through the scalar curvature τ.

G. HARMONIC MEASURE OF A SMALL GEODESIC SPHERE

Let (M,g) be an n-dimensional Riemannian manifold with Brownian motion process $(X_t, t > 0)$. According to the Onsager-Machlup formula, the "most probable path" is that of a classical mechanical particle in a conservative force field whose potential energy is one-twelfth the scalar curvature. This suggests that the "most probable path" follows the negative gradient of the scalar curvature. Accordingly we have proved the following formula for surfaces:

$$E_m\, f(X_{T_\varepsilon}) = \int_{-\pi}^{\pi} f(\theta)\ [1 - \varepsilon^3/32 < \nabla K, u_\theta >]\ \omega(d\theta) + O(\varepsilon^4)$$

where ω is normalized Lebesque measure and $<\nabla K, u_\theta> = K_1 \cos\theta + K_2 \sin\theta$. This formula can be compared with the result obtained by Gray and Willmore for the non-stochastic mean value:

$$M_\varepsilon\, f(m) = \int_{-\pi}^{\pi} f(\theta)\ [1 - \varepsilon^3/12 <\nabla K, u_\theta>]\ \omega(d\theta) + O(\varepsilon^4)$$

This gives a new proof of the fact that if the harmonic measure defined by the Brownian motion is equal to the non-stochastic mean value, then the surface has constant curvature, i.e., locally either a sphere, a plane, or a hyperbolic plane.

H. MEAN EXIT TIME FROM A TUBULAR NEIGHBORHOOD

Currently we are studying the exit time of Brownian motion from a tubular neighborhood of a submanifold. In particular the following

questions are of interest:

(a) To what extent is the mean exit time independent of the imbedding of the submanifold?

(b) To what extent can we recover the intrinsic geometry of the submanifold from the mean exit time?

The interest in (a) is the analogy with the Weyl tube formula, which shows that the volume of a tube in Euclidean space is independent of the imbedding. By contrast we have found that the mean exit time may depend on the imbedding, even when we integrate over the submanifold.

The main tool in the proofs is the "perpendicular laplacian," an ordinary differential operator whose analysis provides the first two terms in the asymptotic expansion of the mean exit time. By considering the special case of plane curves, we show that the higher terms of the mean exit time are not given by the perpendicular laplacian, in general.

In the case of hypersurfaces the method requires no further conditions for its validity. For submanifolds of higher codimension it is required that the volume factor be independent of the normal direction. This is satisfied in particular for Kahler submanifolds of a complex manifold with constant holomorphic sectional curvature.

In case the volume factor is independent of both the base point and the normal direction we obtain an explicit integral formula for the mean exit time. This yields explicit results in the case of hypersurfaces with constant principal curvatures and certain Kahler hypersurfaces, where the integrals can further be expressed in terms of Chern forms.

These results can be specialized to characterize an imbedded hypersphere in various dimensions. One may first note that for an n-dimensional sphere of radius R, the mean exit time from an ε-tubular neighborhood

is $\varepsilon^2/2 + (2-n)\varepsilon^4/24 + 0(\varepsilon^6)$ and that the error term is identically zero when $n = 2$. The general methodology described above is now specialized to prove that when $n = 1$ or 2, the sphere is characterized by the first two terms of the above asymptotic expansion. This hinges on the discovery of a remarkable new quadratic curvature invariant, which is positive definite if $n = 1$, positive semi-definite when $n = 2$, and indefinite when $n \geq 3$. The smallest eigenvalue of the associated quadratic form corresponds to surfaces with all principal curvatures equal, which are known to be spheres.

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This is an account of the work accomplished between 1 June 1983 and 31 May 1984. Work is reported on eight topics: deformation of solids, reliability, seminar on stochastic processes, text on stochastic processes, stability of dependent random variables, mean esit time of Brownian motion with drift, harmonic measure of a small geodesic sphere, and mean exit time for a tubular neighborhood.

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